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Fourier series for synthesis and analysis of periodic signals

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Abstract. This report mainly explores three major parts, namely, periodic discrete-time signals, generation of periodic Fourier signals and analysis of quasi-periodic Fourier signals. We use Matlab to analyze the signal and generate Excel spreadsheet. The waveforms of different frequencies are drawn by Matlab, and the waveforms are analyzed. In addition, the formula for synthesizing periodic signals is simplified and demonstrated here.

Keywords: Fourier series, waveforms, Matlab, spreadsheet, periodic signals, signal analysis.

1. Introduction

For all stages, select appropriate ranges (time, frequency etc.) for the plots to show the appropriate features of the signals and spectra. Label both axes on all plots. Throughout this assignment, the sampling frequency for all signals should be 100 kHz. Generate a set of discrete-time sinusoids using both Excel and MATLAB. The signals should have a specified amplitude and phase (your choices, you may decide to experiment with these), and a frequency as specified below. Plot the sample values against time and observe the resulting waveform. Use a suitable time range so as to show the relevant features of the signal. [1]

2. Generation of periodic discrete time signal

We use MATLAB to generate a set of discrete-time sinusoids. Sinusoidal frequencies of signals include 1 kHz, 5 kHz, 45 kHz, 55 kHz, 95 kHz and 99 kHz. We set the phase of the function as 3 and the amplitude as 5 to draw the relationship between sample value and time as shown in Figure 1. Set the time range from 0 to 0.001, frequency from left to right, from top to bottom is 1KHz, 99KHz, 45KHz, 55KHz, 95KHz, 5KHz. Looking at Figure 1, it can be concluded that the 1KHz discrete sine image can be obtained by rotating 180 degrees from the 99KHz discrete sine image through the horizontal axis (time axis), which is the same as expected. the 1KHz function diagram has an amplitude of 5 and a period of 0.001, which is similar to the general sine function image. The 45KHz discrete sine image is also obtained by rotating the 55KHz discrete sine image by 180 degrees on the time axis. what is different from the expected is that the 45KHz and 55KHz discrete function images are quite different from the expected sine function images. the amplitude of the function is 5, while the period changes to 0.0002, and the oscillation is very severe. Similarly, the 95KHz discrete sine image and the 5KHz discrete sine function image are symmetrical about the horizontal axis (time axis), and are the same as expected. The 95KHz and 5KHz discrete sine images are similar to the general discrete sine function



images, with a period of 0.0002 and an amplitude of 5. However, the periods of 5KHz and 99KHz are five times that of 1KHz and 99KHz, which makes the oscillation more obvious.

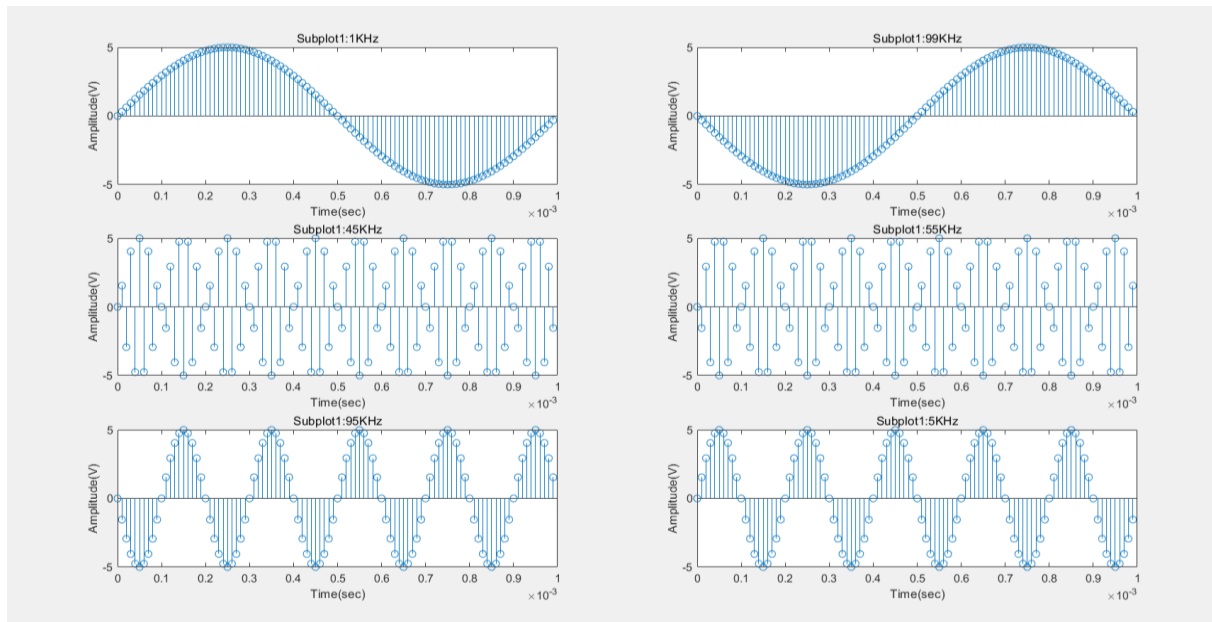


Figure 1. 1khz, 5khz, 45khz, 55khz, 95khz, 99khz discrete sinusoidal images

Then, we import two sets of data from Matlab into Excel, and use Excel to make a 1KHz sine function image (Figure 2), which is found to be consistent with the image in MATLAB. The image shapes of 1KHz and 99Kz are the same because the frequency spectra at these two frequencies are the same. [2]

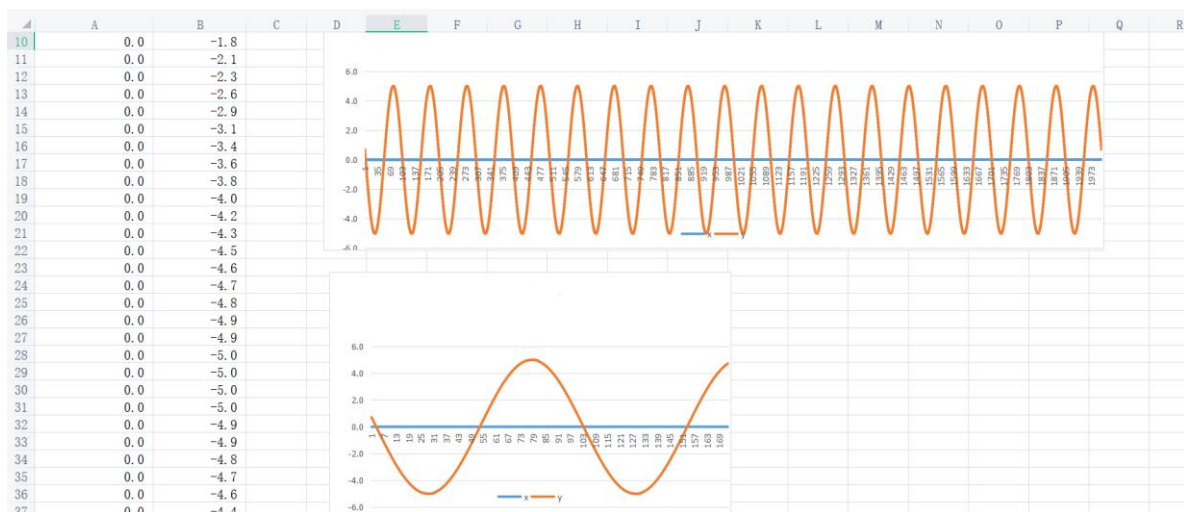


Figure 2. Sinusoidal function diagram with frequency of 1KHz made by Excel

3. Fourier synthesis of periodic signals

3.1. Signal synthesis

First, we analyze the welfare of the function, then simplify it, and import the simplified formula into MATLAB to get a set of data, and export the data to get a spreadsheet. Observing the harmonic function in the topic, it is found that the fundamental frequency parameters a_k , b_k and a_0 have been given, which

are 250Hz, 0.9^k , $1/(2 \cdot k \cdot \pi)$ and 15V, respectively. It is observed that the fundamental wave function consists of a cyclic sum term and an integer term, so I use the "for" cycle to find the function $f(t)$, and finally draw the image of the signal with a spreadsheet (Figure 3). [3]

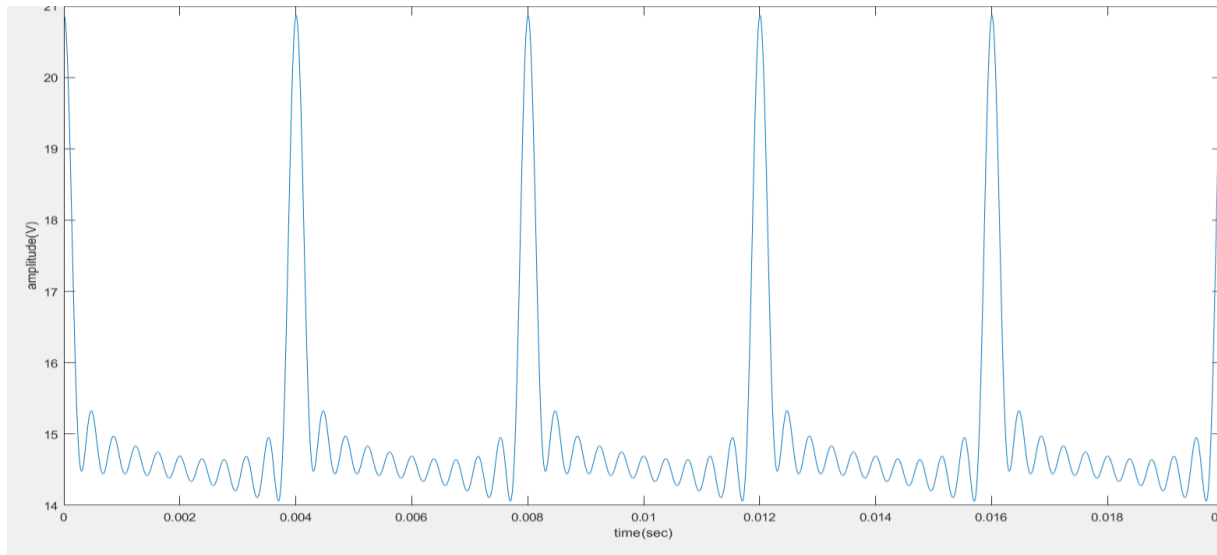


Figure 3. Signal diagram synthesized using spreadsheet

3.2. Formula simplification

According to the analysis topic, knowing the square wave with the function amplitude of 10 V and the fundamental frequency of 250 Hz, it is determined that the formulas of a_0 , a_k and b_k should be determined by Fourier series analysis. [4]

$$f(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos k\omega_0 t + b_k \sin k\omega_0 t)$$

$$\text{We make } x(t) = a_0 + \sum_{k=2}^{\infty} a_k \cos(2\pi k f t) + \sum_{k=2}^{\infty} b_k \sin(2\pi k f t) + a_1 \cos(2\pi f t) + b_1 \sin(2\pi f t)$$

$$\text{Course } T = \frac{1}{f} = 2\pi, \text{ So when } k=1, x(t) = a_0 + a_1 \cos(2\pi f t) + b_1 \sin(2\pi f t)$$

It is equal to

$$\Rightarrow x(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos(2\pi k f t) + b_k \sin(2\pi k f t))$$

$$\Rightarrow x(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos k t + b_k \sin k t)$$

By using Fourier series coefficient formula, we can get

$$a_0 = \frac{2}{\pi} \int_{-\pi}^{\pi} x(t) dt = \frac{2}{\pi} \int_{-\pi}^{\pi} a_0 + \sum_{k=1}^{\infty} (a_k \cos k t + b_k \sin k t) dx$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cdot \cos k x \cdot dx \quad (k=0, 1, 2, 3, \dots)$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cdot \sin k x \cdot dx \quad (k=0, 1, 2, 3, \dots)$$

4. Fourier analysis of quasi-periodic signal

From the 2000 sets of data in the spreadsheet above blackboard, we can fit the functional expressions of the signals we need and draw the images of this set of signals. (Figure 4) Then, Fourier analysis can be carried out on the signal, and the amplitude and phase of 20 frequency components can be obtained by formula conversion operation (Figure 5)

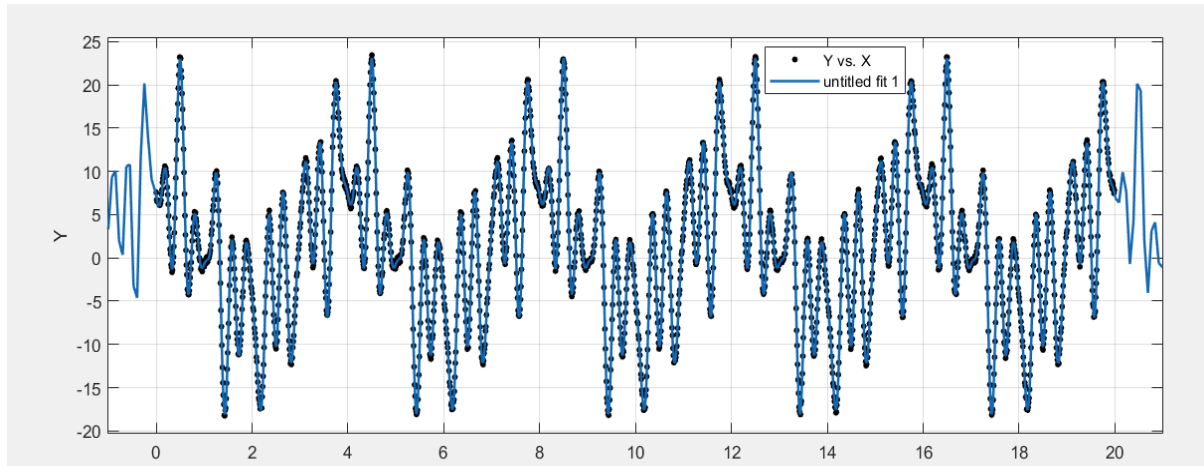


Figure 4. Signal diagram synthesized by Matlab

	1	2	3		1	2	3
1	0			1	1.3007		
2	12.1173		phase position	2	1.4044		amplitude
3	23.5245			3	1.8316		
4	32.7424			4	3.4935		
5	-145.1451			5	70.8371		
6	-170.6433			6	82.2871		
7	-33.8483			7	1.2925		
8	-38.1543			8	0.7504		
9	-28.7227			9	0.5780		
10	-15.0841			10	0.5034		
11	0			11	0.4817		
12	15.0841			12	0.5034		
13	28.7227			13	0.5780		
14	38.1543			14	0.7504		
15	33.8483			15	1.2925		
16	170.6433			16	82.2817		
17	145.1451			17	70.8371		
18	-32.7424			18	3.4935		
19	-23.5245			19	1.8316		
20	-12.1173			20	1.4044		

Figure 5. The amplitude and phase of 20 groups of data obtained by analysing the data

By analysing the phase and amplitude, we can get that the four main AC frequency components in the DC component of the signal are 131.7535, 131.6653, 130.2656 and 127.6100 respectively.

After subtracting the existing synthetic signal from the original signal, we intercepted the signal image of half a period, and found that the number of oscillations of the image was less than that of the original signal image, and the signal image became a little flat. Use the obtained data to generate a spreadsheet, and make a difference with the original signal to get a new set of data, and use this set of data to draw the difference signal. (Figure 6) [5]

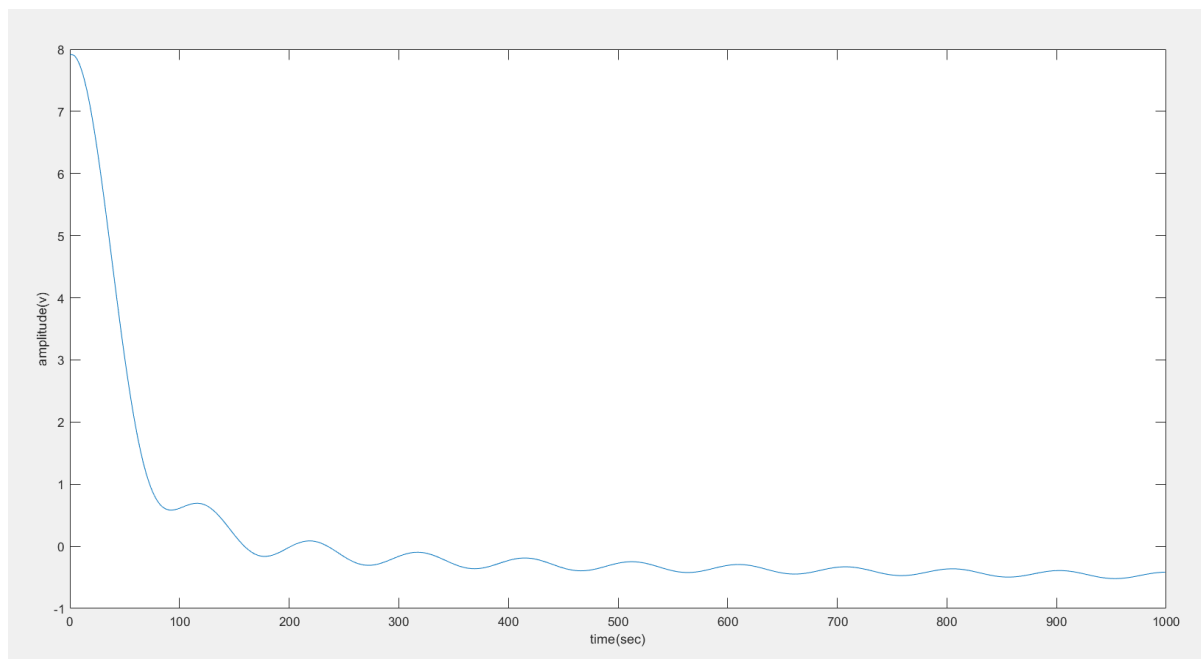


Figure 6. Original image MINUS composite image

The RMS value of the differential signal is obtained by averaging the data extracted from the differential signal, and the RMS value here is 3.873.

5. Conclusion

The concise chart clearly presents the waveform in Excel and Matlab. Describe and comment the observed waveform appropriately, correspond to the expected waveform, and explain the observed phenomenon. The calculation process of synthetic signal is demonstrated concisely and accurately, and the formula can be quoted appropriately when necessary. Give a concise and accurate demonstration of the calculation process of a given signal/waveform. Accurate description of calculation results. A concise chart of the synthesized signal is presented in Excel and Matlab.

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